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DETERMINING ATMOSPHERIC CONDITIONS FROM TRAJECTORY DATA

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13. ABSTRACT (Maximum 200 words)

This report investigates the feasibility of determining atmospheric conditions from a projectile's observed flight. Test trajectories were generated as solutions of the modified point-mass (MPM) equations of motion, using standard atmospheric conditions and specified winds. In a revised version of the MPM equations, the air density, sound speed and wind were re-defined in terms of a small number of parameters. A curve-fitting program then fit the revised equations to the trajectory data by adjusting the parameter values. Density, sound speed and wind profiles obtained from these fits were in excellent agreement with the original atmospheric data. Thus, the program - although it should be further tested under less idyllic conditions - is entirely feasible.

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Table of Contents

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I. Introduction

This is a report on the feasibility of determining atmospheric conditions from a projectile's observed trajectory. In this preliminary study, 'measurements' are generated as solutions of the modified point-mass (MPM) equations of motion, using the 1962 U.S. Standard Atmosphere and specified wind profiles. The hope is that eventually the generated data will be replaced in the program by instrument readings of trajectory variables.

The over-all scheme is as follows. The MPM equations are altered by re-defining the air density, sound speed and wind in terms of a small number of parameters. A nonlinear curve-fitting technique known as FINLIE [Ref. 1] then fits the revised differential equations to the generated data by adjusting the parameter values. The original and fitted air density, sound speed and wind histories can then be compared.

The FINLIE user must construct a FORTRAN subroutine defining the original set of equations *plus an auxiliary set.* The auxiliary equations involve partial derivatives of the dependent variables of the original set. Expressions for these partial derivatives can be extremely complicated and the opportunity for error - both in deriving the expressions and in coding them - is great. Hence a software package called MACSYMA[1] [Refs. 2 and 3] was used. From the original equations, this versatile program determined expressions for each of the partial derivatives and presented these expression as FORTRAN code.

In the next few sections, we will discuss the modified point-mass equations, the standard atmosphere used, the revised equations used to fit the data, and the application of FINLIE/MACSYMA to the fitting problem. Succeeding sections will discuss results and conclusions.

II. The Modified Point-Mass Equations

The modified point-mass trajectory model is the primary method of simulating trajectories in the preparation of Firing Tables. The basic equation of motion, derived in Ref. 4, can be re-written [Ref. 5] in the form

$$\boxed{\dot{\vec{U}} = (D - A)\vec{V} + \vec{G}_A} \tag{1}$$

where

$$\begin{aligned} D &= \frac{h_a(\vec{G} \bullet \vec{V})}{(1 + h_a)V^2} \\ A &= \left(\frac{\rho S \ell}{2m} C_D\right)\left(\frac{V}{\ell}\right) \\ \vec{G}_A &= \frac{1}{1 + h_a}\left[\vec{G} + \frac{h_L(\vec{G} \times \vec{V})}{(1 - h_M)V}\right] \end{aligned}$$

[1] MACSYMA is a trademark of Symbolics, Inc., 257 Vassar St., Cambridge, MA 02139

$$
\begin{aligned}
h_L &= k_a^2\left(\frac{C_{L_\alpha}}{C_{M_\alpha}}\right)\left(\frac{\dot\phi\ell}{V}\right)\\
h_M &= k_a^2\left(\frac{C_{N_{p\alpha}}}{C_{M_\alpha}}\right)\left(\frac{\dot\phi\ell}{V}\right)^2\\
h_a &= \frac{h_L^2}{1-h_M} - h_M
\end{aligned}
$$

and where

$\vec{U}$ = projectile velocity with respect to the earth

$\vec{V} = \vec{U} - \vec{W}$ = projectile velocity with respect to the air

$\vec{W}$ = wind velocity with respect to the earth

$\vec{G}$ = the sum of the gravity and Coriolis accelerations

$\dot\phi$ = axial spin, rad/s.

(All symbols are defined in the List of Symbols.)

The axial spin $\dot\phi$ is obtained as the solution of a simplified roll equation:

$$\ddot\phi = -BC_{\ell_p}\left(\frac{\dot\phi\ell}{V}\right) \tag{2}$$

where

$$B = k_a^{-2}\left(\frac{\rho S\ell}{2m}\right)\left(\frac{V}{\ell}\right)^2.$$

The aerodynamic coefficients C_{L_α}, C_{M_α}, $C_{N_{p\alpha}}$, and C_{ℓ_p} in Eqs.(1) and (2) are assumed to be functions of Mach number. The drag coefficient C_D is assumed to depend on both Mach number and the yaw of repose, $\vec{\alpha}_e$. In particular, C_D is assumed to have the form

$$C_D = C_{Do} + C_{D2}|\vec{\alpha}_e|^2 \tag{3}$$

where C_{Do} is a function of Mach number and C_{D2} is a constant.

The yaw of repose can be computed from the relation [Ref. 5]:

$$\vec{\alpha}_e = \frac{\dot\phi\,\vec{G}_A \times \vec{V}}{BV^2C_{M_\alpha}}. \tag{4}$$

III. Component Form

The vector equation (1) is not in a suitable form for programming; we need component expressions. Hence our first task is to select a convenient coordinate system for describing the motion of a projectile along its trajectory.

For the moment, assume that the launch point is at some sea-level spot on the earth's surface. Then we set our origin at the launch point and define a right-handed Cartesian system as follows: the 1- and 3-axes form a plane tangent to the earth at the origin; the 2-axis is perpendicular to this plane, positive upward, and the 1-axis is chosen so that the velocity $\vec{U}$ at time zero is in the 1-2 plane. We will call this coordinate system the 'flat-earth' system. Then the projectile's position vector $\vec{X}$ with respect to the earth can be written in component form as

$$\vec{X} = (X_1,\ X_2,\ X_3)$$

where

X_1 is the down-range distance,

X_2 is the height above the 1-3 plane,

X_3 is the lateral distance, positive to the right when looking down-range,

and where $\dot{\vec{X}} = \vec{U}$. Since the trajectory – in most cases – lies nearly in the 1-2 plane, X_3 is usually much smaller in magnitude than X_1 and X_2.

Similarly, we can write

$$\begin{aligned}
\vec{U} &= (U_1,\ U_2,\ U_3) \\
\vec{V} &= (V_1,\ V_2,\ V_3) \\
\vec{W} &= (W_1,\ W_2,\ W_3) \\
\vec{G} &= (G_1,\ G_2,\ G_3).
\end{aligned}$$

where U_3 is usually much smaller in magnitude than U_1 and U_2. The initial velocity is given by

$$\vec{U}_o = |\vec{U}_o|(\cos E,\ \sin E,\ 0)$$

where E is the gun elevation.

In the system we have just described, the launch point is at (0,0,0). However, if the launch point is not at sea level (for example, if it is half-way up a mountain), then we choose our origin at the sea-level point directly beneath the launch point. Thus the origin of our system may be under the earth's surface. In general, the launch point is at $(0,\ X_{20},\ 0)$, where X_{20} is the height of the launch point above sea level. The motivation for this rather cumbersome shifting of the origin is to retain X_2 as a measure of the height above sea level for a flat-earth model.

Eq.(1) can be written in component form as

$$\dot{U}_1 = (D - A)V_1 + \frac{1}{1+h_a}\left[G_1 + \frac{h_L(G_2V_3 - G_3V_2)}{(1-h_M)V}\right] \qquad (5)$$

$$\dot{U}_2 = (D - A)V_2 + \frac{1}{1+h_a}\left[G_2 + \frac{h_L(G_3V_1 - G_1V_3)}{(1-h_M)V}\right] \qquad (6)$$

$$\dot{U}_3 = (D - A)V_3 + \frac{1}{1+h_a}\left[G_3 + \frac{h_L(G_1V_2 - G_2V_1)}{(1-h_M)V}\right]. \qquad (7)$$

Before we can program these equations, we need expressions for the components of $\vec{G}$. Recall that $\vec{G}$ is the sum of the gravity and Coriolis accelerations:

$$\vec{G} = \vec{g} + \vec{C} \tag{8}$$

The gravity vector can be written as

$$\vec{g} = -\frac{g_0(X_1/R,\ 1 + X_2/R,\ X_3/R)}{[(X_1/R)^2\ +\ (1 + X_2/R)^2\ +\ (X_3/R)^2]^{3/2}} \tag{9}$$

where

g_0 = magnitude of $\vec{g}$ at the origin (the standard value at 45 deg. N. latitude being 9.80665 m/s^2)

R = 'effective' radius of the earth (6 356 766 m).

This expression for $\vec{g}$ is usually simplified by ignoring X_3/R and assuming that X_1/R and X_2/R – while not ignorable – are much less than unity. Then we have

$$\begin{aligned}\vec{g} &\approx -\frac{g_0(X_1/R,\ 1 + X_2/R,\ 0)}{(1 + X_2/R)^3} \\ &\approx -g_0(X_1/R,\ (1 + X_2/R)^{-2},\ 0) \\ &\approx -g_0(X_1/R,\ 1 - 2X_2/R,\ 0)\end{aligned} \tag{10}$$

The Coriolis acceleration is given by

$$\vec{C} = -2\vec{\omega} \times \vec{U} \tag{11}$$

where $\vec{\omega}$ is the earth's angular velocity vector:

$$\begin{aligned}\vec{\omega} &= (\omega_1,\ \omega_2,\ \omega_3) \\ &= \omega_E(\cos L\ \cos AZ,\ \sin L,\ -\cos L\ \sin AZ)\end{aligned} \tag{12}$$

where

$$\omega_E = |\vec{\omega}| = \frac{2\pi\ rad/sidereal\ day}{86164.09\ s/sidereal\ day} \approx 7.291 \times 10^{-5}\ rad/s$$

and where

L = latitude at launch point (for the Southern Hemisphere, replace L by -L).

AZ = azimuth of the 1-axis, measured clockwise from North.

Hence we have

$$G_1 = -g_0 X_1/R \qquad - 2(\omega_2 U_3 - \omega_3 U_2) \tag{13}$$
$$G_2 = -g_0(1 - 2X_2/R) - 2(\omega_3 U_1 - \omega_1 U_3) \tag{14}$$
$$G_3 = -2(\omega_1 U_2 - \omega_2 U_1) \tag{15}$$

IV. Atmospheric Conditions

The equations of motion used in this investigation are essentially Eqs.(5-7) and (2). However, two versions of this set of equations had to be programmed: one to generate data (X_1, X_2, and X_3 *vs.* t) and one to fit the generated data.

In the generating version, the wind, air density, and speed of sound are somewhat complicated functions of altitude. We will discuss these functions shortly.

In the fitting version, simpler expressions are assumed for the wind, air density, and sound speed. We will see later that these expressions involve just five fitting parameters. The whole point of this exercise is to fit the second version of the equations to the given trajectory (probably piecewise) to obtain values for those five parameters.

1. Atmosphere Used in the Generating Equations

For purposes of generating test trajectories, we used the air density and speed of sound from the *U.S. Standard Atmosphere, 1962* [Ref. 6], hereafter called STAT. The 'geometric' altitude Z discussed in STAT is not quite the same as X_2, the height above the 1-3 plane in our flat-earth system. A better approximation to the altitude allows for the curvature of the earth. We have[1]

$$(R + X_2)^2 + X_1^2 = (R + Z)^2$$

so that

$$\begin{aligned} Z &= \sqrt{(R + X_2)^2 + X_1^2} - R \\ &\approx X_2 + X_1^2/2R \end{aligned} \qquad (16)$$

Note that in approximation (16), the difference between Z and X_2 is solely a function of the down-range distance X_1. At $X_1 = 25$ kilometers, for example, $Z - X_2$ is about 50 metres for any value of X_2. If a trajectory is to be run to impact at ground level, we should set $X_2 = -X_1^2/2R$, not $X_2 = 0$, as the stopping point.

The STAT value for g, the magnitude of $\vec{g}$, is given by

$$g(Z) = \frac{g_0}{(1 + Z/R)^2} \qquad (17)$$

In the STAT system, the basic altitude (up to $Z = 90$ km) is the 'geopotential' altitude H, defined as

$$H = \int_0^Z \frac{g(s)\, ds}{g_0} = \frac{Z}{1 + Z/R}. \qquad (18)$$

STAT divides altitude H into eight zones, in each of which the temperature is assumed to be linear in H:

$$T = T_B + T'(H - H_B) \qquad (deg\ K) \qquad (19)$$

[1] Strictly, the geometric altitude Z is measured along a line of gravitational force; however, the distinction between this line of force and a straight line is negligible for our purposes.

Table 1. 1962 US Standard Atmosphere Values

Zone	H (km)	H_B (km)	T_B (deg K)	T' (deg K/km)	ρ_B (kg/m^3)
1	-5, 11	0	288.15	-6.5	1.225000+0
2	11, 20	11	216.65	0	3.6391803-1
3	20, 32	20	216.65	1	8.8034864-2
4	32, 47	32	228.65	2.8	1.3225009-2
5	47, 52	47	270.65	0	1.4275335-3
6	52, 61	52	270.65	-2	7.5943220-4
7	61, 79	61	252.65	-4	2.5109063-4
8	79, 89	79	180.65	0	2.0011372-5

where T_B is the value of T at H_B, the bottom altitude for the zone (except in the lowest zone[1], where $H_B = 0$ but the zone extends down to -5 kilometres). Values of H_B, T_B, and the slope T' ($= dT/dH$) are given in Table 1 for each of the eight zones.

The speed of sound at a given altitude can be computed from the corresponding temperature (deg K):

$$V_s = 20.04680276\sqrt{T} \qquad (m/s) \tag{20}$$

For those zones in which T' is not zero, the STAT air density is given by

$$\frac{\rho}{\rho_B} = \left(\frac{T}{T_B}\right)^{-1-Q/T'} \tag{21}$$

where

$$Q = \frac{g_0 * molecular\ weight}{universal\ gas\ constant} \approx 34.16319474 \quad (deg\ K/km)$$

and where, as before, subscript B denotes the value at $H = H_B$.

For those zones in which $T' = 0$ (that is, in which the temperature is assumed constant), the STAT air density is given by

$$\rho/\rho_B = e^{-Q(H-H_B)/T_B} \tag{22}$$

We obtained the approximate values of ρ_B listed in Table 1 by setting $\rho_B = 1.225$ kg/m^3 for zone 1 and then calculating ρ_B for each succeeding zone as the density at the top of the previous zone.

There is no 'standard' wind profile; hence the winds used in generating trajectories were arbitrarily chosen as follows: the vertical wind component, W_2, was zero while W_1 and W_3 were specified quadratics in height X_2.

[1] "Weather", as we usually think of it (clouds, rain, snow, etc.) is almost always confined - like most of us - to zone 1, the troposphere. Although the nominal rate of temperature decrease in zone 1 is 6.5 deg K/km, local temperature inversions are common.

2. Atmosphere Used in the Fitting Equations

Eventually (but not in this preliminary study) we intend to divide the trajectory into overlapping time intervals, some arbitrary time within one interval (say, the mid-time) serving as the initial time for the next segment. (These time intervals are not to be confused with STAT altitude zones; a time interval could lie entirely in one altitude zone or involve two or more zones.)

For the fitting equations, we decided that over any segment of the trajectory the wind could be approximated by the relations

$$W_1 = C_1 + C_2 X_2 \tag{23}$$
$$W_2 = 0 \tag{24}$$
$$W_3 = C_3 + C_4 X_2 \tag{25}$$

This gave us four parameters (so far) to be determined by the fit: C_1 through C_4. To keep the total number of parameters to a minimum (for reasons that will become clearer when we discuss FINLIE), we decided to express the speed of sound and the air density in terms of a single parameter:

$$C_5 = T' \tag{26}$$

The speed of sound is assumed to have the form of Eq.(20):

$$\boxed{V_s = 20.04680276\,\sqrt{T_A + C_5(H - H_A)}} \qquad (m/s) \tag{27}$$

where T_A and H_A – the temperature and altitude at the starting time of the interval under consideration – are assumed to be *known* quantities.

The density relation (21) is unsuitable for fitting purposes because it tends to become indeterminate as T' ($= C_5$) goes to zero. Actually, in an earlier phase of this investigation, we did use Eq.(21) to fit a generated trajectory that remained in zone 1; that is, for which the altitude was less than 11 km. Starting from a non-zero estimate for C_5, FINLIE zeroed in on the correct value (-6.5 deg K/km) in a few iterations. However, we can't in general be sure that C_5 will be nonzero in the fitting interval and we certainly don't want to preclude an initial estimate of zero. Hence we decided to re-write Eq.(21) so as to remove the indeterminacy. To do so, we set

$$\frac{T}{T_B} = \frac{T_B + T'(H - H_B)}{T_B} = 1 + \Delta$$

where

$$\Delta \equiv \frac{T'(H - H_B)}{T_B}.$$

We see from Table 1 that in any zone, $|\Delta|$ is less than 1; hence we can express $\ln(1 + \Delta)$ as a convergent power series:

$$\ln(T/T_B) = \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \cdots.$$

Eq.(21) can then be written as

$$\begin{aligned}
\frac{\rho}{\rho_B} &= exp[-(1+Q/T')\ln(T/T_B)] \\
&= exp\left[-(1+Q/T')\Delta\left(1-\frac{\Delta}{2}+\frac{\Delta^2}{3}-\frac{\Delta^3}{4}+\cdots\right)\right] \\
&= exp\left[-\frac{(T'+Q)(H-H_B)}{T_B}\left(1-\frac{\Delta}{2}+\frac{\Delta^2}{3}-\frac{\Delta^3}{4}+\cdots\right)\right]. \qquad (28)
\end{aligned}$$

This form of the STAT density expression (21) has the computational advantage that it offers no difficulties as T' goes to zero; in fact, at $T'=0$, the new form reduces to Eq.(22). In our fitting equations, we will eventually use a truncated form of Eq.(28):

$$\boxed{\rho/\rho_A = exp[-(C_5+Q)(H-H_A)/T_A]} \qquad (29)$$

where ρ_A – the value of ρ at the start of the fitting interval – is assumed to be known. For this preliminary study, we used the expanded form (28) in order to recover – if possible – the generated value of T'.

Only the first time-interval values of T_A, H_A, and ρ_A (presumably obtained in 'real life' by measurements at the launch site) are required inputs to the fitting process: thereafter, the closing values for the k-th interval become the starting values for the (k+1)-th interval.

V. FINLIE

FINLIE [Ref. 1] is a FORTRAN program for fitting a system of first-order ordinary differential equations to measurements taken on one or more of the dependent variables. No knowledge of the system's solution is required; indeed, in most cases, no closed-form solution exists.

Our first step in using FINLIE was to re-write the equations in the required form. We set

$$\begin{aligned}
y_1 &= X_1 & \qquad y_4 &= \dot{X}_1 = U_1 \\
y_2 &= X_2 & y_5 &= \dot{X}_2 = U_2 \\
y_3 &= X_3 & y_6 &= \dot{X}_3 = U_3 \\
& & y_7 &= \dot{\phi}
\end{aligned}$$

so that Eqs.(5-7) and (2) could be expressed as a system of seven first-order equations:

$$\begin{aligned}
\dot{y}_1 &= F_1\ [= y_4] & \qquad \dot{y}_4 &= F_4\ [= RHS\ of\ Eq.(5)] \\
\dot{y}_2 &= F_2\ [= y_5] & \dot{y}_5 &= F_5\ [= RHS\ of\ Eq.(6)] \\
\dot{y}_3 &= F_3\ [= y_6] & \dot{y}_6 &= F_6\ [= RHS\ of\ Eq.(7)] \qquad (30) \\
& & \dot{y}_7 &= F_7\ [= -BC_{\ell_p}(y_7\ell/V)]
\end{aligned}$$

The FINLIE user must construct a FORTRAN subroutine defining the system of equations – in our case, system (30) – and an auxiliary set of so-called influence equations.

To describe the latter set, we introduce q_k where

$$\begin{aligned} q_1, \dots, q_7 &= y_{10}, \dots, y_{70} \\ q_8, \dots, q_{12} &= C_1, \dots, C_5 \end{aligned}$$

Influence coefficients, the dependent variables of the influence equations, are defined as

$$D_{jk} \equiv \frac{\partial y_j}{\partial q_k} \tag{31}$$

where index j runs from 1 to n and index k runs from 1 to (n + N), n being the order of the original system (7 in our case) and N the number of parameters (here 5). Then the influence equations can be written in the form [Ref. 1]:

$$\dot{D}_{jk} = \sum_{i=1}^{n} \left(\frac{\partial F_j}{\partial y_i} \right) D_{ik} + C_{jk} \tag{32}$$

where

$$\begin{aligned} C_{jk} &= 0 && if \;\; k \leq n \\ &= \frac{\partial F_j}{\partial C_{k-n}} && if \;\; k > n \end{aligned}$$

FINLIE automatically assigns the proper initial conditions to the influence equations [$(D_{jk})_o$ is 1 if j = k and 0 otherwise] and integrates the influence equations simultaneously with the original system.

Note that there are n(n + N) influence equations. This can be an uncomfortably large number. We can't usually do anything about n but it helps if we can keep N, the number of parameters, as small as possible. This is why we restricted ourselves to 5 parameters and 'only' 7(7 + 5) = 84 influence equations.

Because of the simplicity of the first three of the seven equations in set (30), the corresponding influence equations are equally simple:

$$\dot{D}_{1k} = D_{4k} \tag{33}$$

$$\dot{D}_{2k} = D_{5k} \tag{34}$$

$$\dot{D}_{3k} = D_{6k} \tag{35}$$

The remaining four/seventh of set (32) is not quite so easily expressed. Consider, for example, just one of the partial derivatives needed to evaluate $\dot{D}_{4k}$, say

$$\frac{\partial F_4}{\partial y_2} = \frac{\partial F_4}{\partial X_2}$$

where F_4 is the right-hand side of Eq.(5). Note that X_2 enters F_4 in three ways:

- through the gravity component G_2, Eq.(14);
- through the geometric altitude Z, Eq.(16), and from there into H and ρ and V_s;

- through the winds, W_1 and W_3 [Eqs.(23) and (25)], and from there into velocity V.

And, of course, since the aerodynamic coefficients are specified functions of Mach number ($= V/V_s$), X_2 is involved wherever there is an aerodynamic coefficient. Thus determining an expression for $\partial F_4/\partial X_2$ – or indeed for many of the other partials – is not a simple task.

Two factors came to our aid at this point. First, we were able to make some simplifying assumptions. As Ref. 1 points out,

> ... certain liberties can be taken with the influence equations: expressions can be approximated by simpler ones, the effect of certain [parameters] on certain terms in the original equations can be ignored, etc. If done with care and judgment, such simplifications will have no effect on the final answer: the same [fit] will be reached with or without the simplifications.

In our case, the simplifications concern the yaw of repose. A close study of Eq.(4) reveals that $\vec{\alpha}_e$ is an impossibly intricate function of six of the seven dependent variables (all but X_3) and all five parameters C_i. The magnitude $|\vec{\alpha}_e|$ is usually small and only its square appears explicitly (in Eq.(3), where it constitutes a relatively minor addendum to the drag coefficient). We kept $|\vec{\alpha}_e|^2$ in our basic fitting equations, of course, but we decided – with a fairly clear conscience – to ignore its partial derivatives with respect to the dependent variables and the parameters. It just wasn't worth the formidable effort involved in adding those complexities to already labyrinthine expressions.

The second factor that came to our aid was the availability of an automatic derivative-taker: MACSYMA. This software package is discussed in the next section.

VI. MACSYMA

MACSYMA (project MAC's SYmbolic MAnipulation) is an interactive expert system – written in LISP – for performing symbolic *and* numerical operations. As elementary examples (from Ref. 3):

- if asked for $d(sin\ x)/dx$, MACSYMA answers: $cos(x)$,
- if asked for $\int sin(x)\ dx$, MACSYMA answers: $-cos(x)$,
- if asked to 'simplify' $sin^5 x$, MACSYMA answers:

$$\frac{sin(5x)}{16} - \frac{5sin(3x)}{16} + \frac{sin(x)}{8}$$

MACSYMA can expand a given function in a Taylor or Laurent series, invert a matrix of symbols, solve a system of nonlinear algebraic equations, manipulate tensors, solve differential equations, etc., etc.; the list goes on. A very useful program. In particular, MACSYMA was used by us

- to find expressions for the partial derivatives of F_j with respect to the seven dependent variables and five fitting parameters, as required in Eq.(32),
- to convert those expressions into FORTRAN statements for insertion in the subroutine required by FINLIE.

Construction of this subroutine is the only really challenging part of using FINLIE and forming the partial derivative expressions is the only challenging part of constructing the subroutine. Thus we see that MACSYMA played a key role in the coding of our program.

We will not attempt a short course in the use of MACSYMA (see, instead, Refs 2 and 3). Two points, however, are worthy of note.

Firstly, it would be helpful if MACSYMA could – on its own – recognize recurring expressions and take appropriate short-cuts. We have found that its abilities in this regard are limited (this may be due more to our inexperience with MACSYMA than to any inherent shortcomings in the program). However, with a little nudging, those short-cuts can be imposed on MACSYMA. Consider, for example, that ubiquitous V:

$$\begin{aligned} V &= \sqrt{(U_1 - W_1)^2 + (U_2 - W_2)^2 + (U_3 - W_3)^2} \\ &= f(y_2, y_4, y_5, y_6, C_1, C_2, C_3, C_4) \end{aligned}$$

We see that the partials of V with respect to four of the dependent variables and four of the fitting parameters are needed:

$$\frac{\partial V}{\partial y_2} = -\frac{[(y_4 - W_1)C_2 + (y_6 - W_3)C_4]}{V}$$

and so on. We have found that if MACSYMA is not told differently, it will re-derive such expressions every time V is encountered. The resulting FORTRAN expressions for the partials of the F_j's could fill pages of print-out. To prevent this, we introduced dummy symbols for these repeating expressions (say, DVY2 for $\partial V/\partial y_2$) and instructed MACSYMA to use the chain rule where applicable:

$$\frac{\partial F_4}{\partial y_2} = \frac{\partial F_4}{\partial V}\, DVY2 + \ldots$$

and so on. This reduced the size of the subroutine considerably and – because the subroutine is called often by FINLIE in its integration routine – it reduced the run time as well.

The second point of interest is the possibility of bugs in the MACSYMA program. According to Rand (Ref. 3), "Any system as large and complicated as MACSYMA is bound to have some bugs in it." After considering the alternatives, we decided to trust MACSYMA.

VII. Test Projectile

To generate trajectories, we had to postulate some projectile, real or hypothetical, with its associated physical and aerodynamic properties. We chose a hypothetical projectile

Table 2. Mach Dependency of the Test Case Aerodynamic Coefficients

Mach	C_{D_o}	Mach	C_{L_α}	Mach	C_{M_α}	Mach	C_{ℓ_p}
0.000	.1394	0.00	1.6568	0.00	4.197	0.0	-.0178
0.800	.1394	1.11	1.6568	0.40	4.271	0.5	-.0145
0.850	.1425	1.60	1.7514	0.80	4.663	1.0	-.0123
0.875	.1507	3.00	2.0584	0.90	5.039	2.0	-.0096
0.900	.1693			0.93	5.364	3.0	-.0079
1.050	.3685			1.00	4.850		
1.075	.3871			1.30	4.570		
1.100	.3954			1.60	4.478		
1.150	.3943			3.00	4.478		
1.300	.3716						
1.500	.3458						
1.700	.3241						
2.100	.2849						
3.000	.2106						

(bearing some resemblance to the M483A1 artillery projectile) with the following physical properties:

$$\begin{aligned} \ell \ (diameter) &= 155 \ mm \\ m \ (mass) &= 46.947 \ kg \\ I_x &= 0.1595 \ kg - m^2 \end{aligned}$$

Of the six aerodynamic coefficients involved in our equations of motion:

$$C_{D_o}, \ C_{D2}, \ C_{L_\alpha}, \ C_{M_\alpha}, \ C_{N_{p\alpha}}, \text{ and } C_{\ell_p},$$

four were selected to be functions of Mach number, as indicated in Table 2. This table shows pairs of values: Mach number and corresponding coefficient value. Our code performs straight-line interpolation for Mach values between two entries. The remaining two aerodynamic coefficients were assumed to be constant:

$$\begin{aligned} C_{D2} &= 4.171 \\ C_{N_{p\alpha}} &= -1.8 \end{aligned}$$

Note that exactly the same aerodynamic behavior was assumed in the fitting equations as was used in the equations for generating the trajectories. It is unlikely that an actual flight would duplicate so precisely a projectile's assumed data base of aerodynamics.

VIII. Test Conditions

For this preliminary study, we chose a rather elementary situation: one in which the entire trajectory could reasonably be contained in a single time interval. To make a single-interval fit feasible, we imposed two conditions on the equations:

(a) the wind components W_1 and W_3 were conveniently chosen to be linear with X_2 over the entire trajectory:

$$\begin{aligned} W_1 &= -6 + 0.002X_2 \ \ m/s \\ W_2 &= 0 \\ W_3 &= 3 + 0.004X_2 \ \ m/s, \end{aligned}$$

where one m/s is a little less than two knots;

(b) the initial velocity (659 m/s) and quadrant elevation (800 mils = 45 deg) were chosen so that the trajectory remained in altitude zone 1.

Thus we see from Table 1 that the temperature was generated as

$$T = 288.15 - C_5 H \qquad\qquad deg\ \ K$$

where $C_5 = -6.5$ deg K/km, and the air density was generated as

$$\rho = 1.225 \left(\frac{T}{288.15}\right)^{-1-Q/C_5} \qquad\qquad kg/m^3$$

Of course, the same initial atmospheric values (288.15 deg K and 1.225 kg/m^3) were input to the fitting equations.

The latitude L for our test run was arbitrarily set at 45 deg North; the azimuth AZ was zero; g_0 was 9.80665 m/s^2. The initial spin was taken to be 1335.383 rad/s.

IX. Test Results

Figures 1, 2, and 3 show the generated X_1, X_2, and X_3 values, respectively, versus time. FINLIE's task was to recover from those prosaic-looking curves the atmospheric conditions: air density, temperature and wind history over the course of the trajectory. (In Figure 3, the lateral motion for zero wind is also shown; in Figures 1 and 2, the no wind curves would lie almost on top of the given curves.)

Table 3 lists the values of three initial conditions and five parameters:

- the column headed "Gen. value" lists the values used to generate those X_1, X_2, and X_3 values plotted in Figures 1 to 3;
- the column headed "Initial est." lists the input estimates used by FINLIE to start the fitting process;
- the column headed "Final fitted value" lists the values obtained by FINLIE in the final iteration of its fitting process.

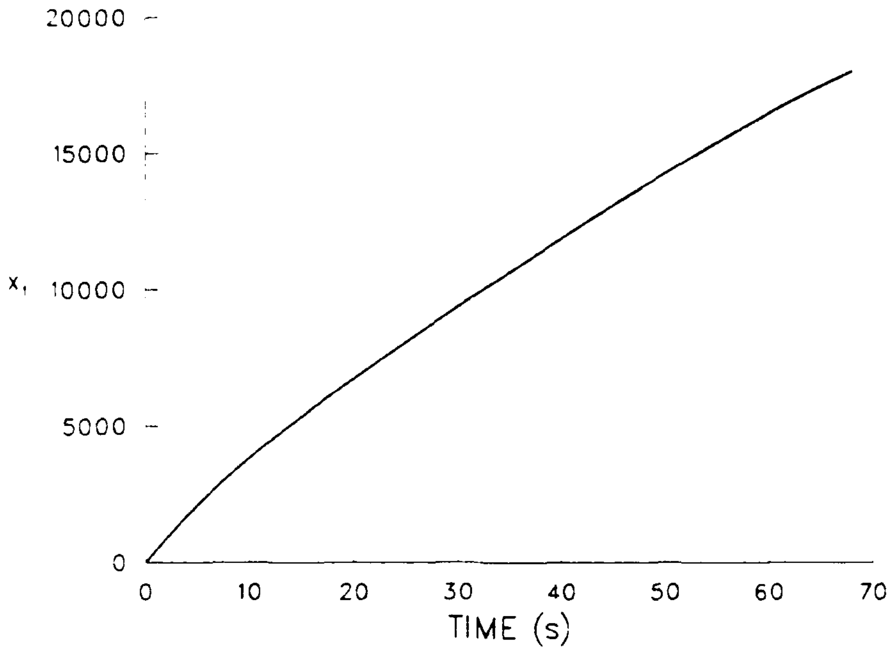

Figure 1. Down-Range Component X_1 vs. time

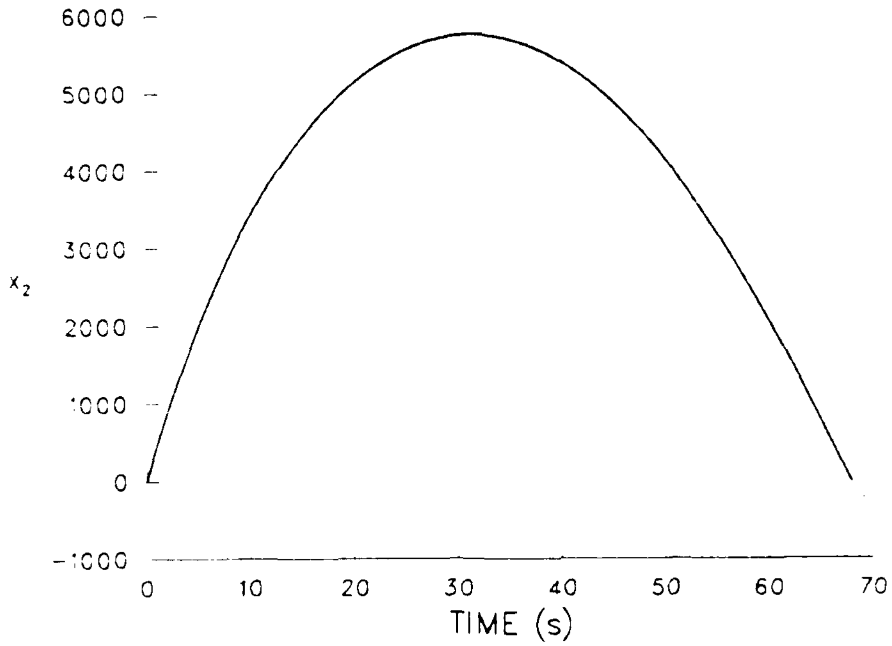

Figure 2. Vertical Component X_2 vs. time

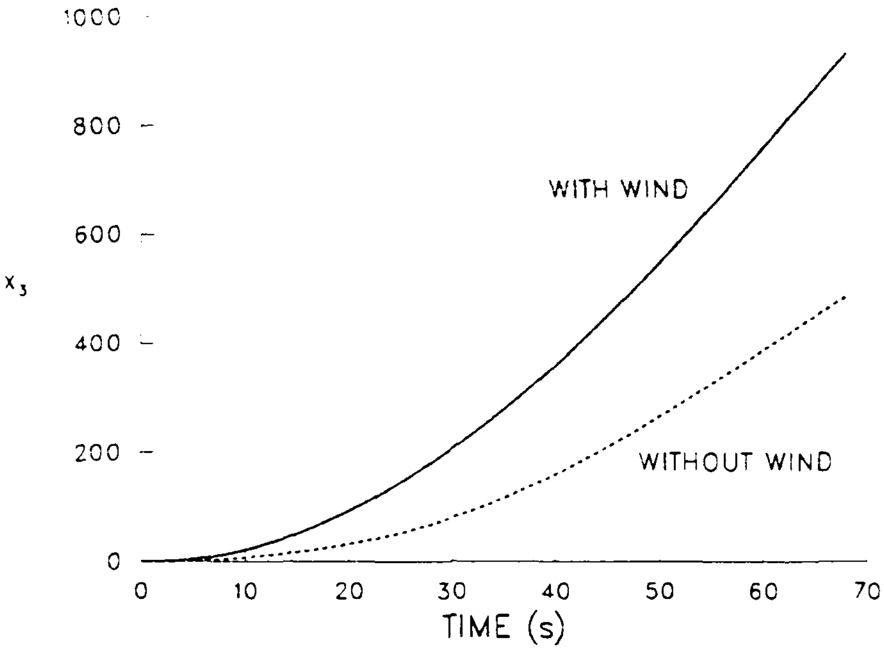

Figure 3. Lateral Component X_3 vs. time

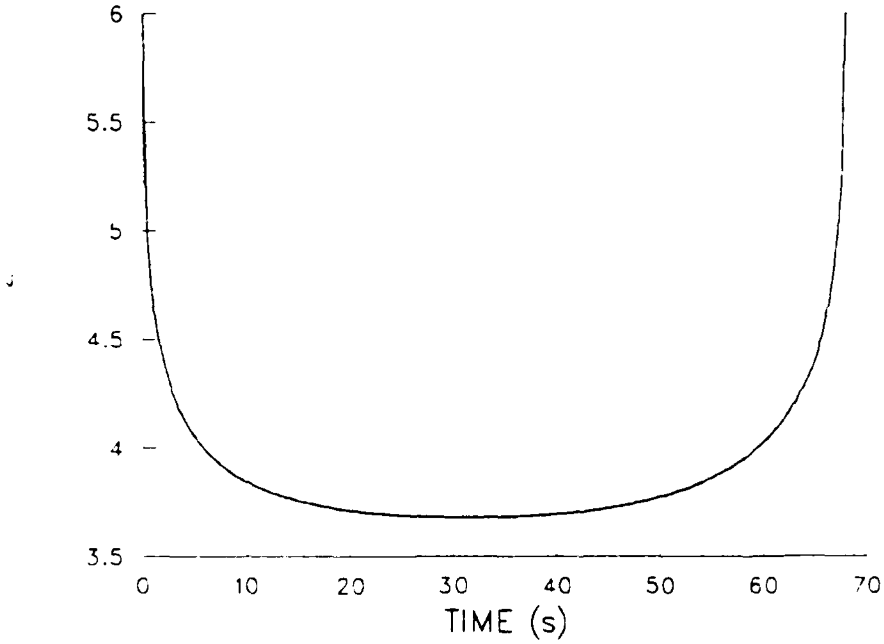

Figure 4. Conservative Estimate of the Significant Digits in the Fitted Air Density

Table 3. Test Case Results

IC and Parameters	Gen. value	Initial est.	Final fitted value
$\dot{X}_{10}$ (m/s)	465.983	463.690	465.975
$\dot{X}_{20}$ (m/s)	465.983	462.745	465.983
$\dot{X}_{30}$ (m/s)	0	0.030	0.003
C_1 (m/s)	-6	4	-5.956
C_2 (1/s)	0.002	0.001	0.00199
C_3 (m/s)	3	-4	2.982
C_4 (1/s)	0.004	0.001	0.00400
C_5 (deg K/km)	-6.5	-0.65	-6.507

(FINLIE can be instructed to fix the value of any initial condition or parameter at its input value. This was done for four of the seven initial conditions: the position vector = (0,0,0) and the initial spin, $\dot{\phi}_0 = 1335.683$ rad/s. We decided to let FINLIE determine the initial velocity components: the first estimates shown in Table 3 were obtained by having the program take position and time differences for the first two data points.)

We see from Table 3 that the final, fitted values are in very close agreement with the values used to generate the trajectory. Note that the initial parameter estimates are deliberately poor: convergence of the fitting process didn't depend on starting close to the answer. Plots of the generated and fitted air densities (ρ_{gen} and ρ_{fit}) would be indistinguishable to the eye. Hence we choose a more informative means of comparison: the number of significant digits in ρ_{fit} compared to ρ_{gen}. Let E_ρ be the relative density error:

$$E_\rho = |(\rho_{gen} - \rho_{fit})/\rho_{gen}|$$

A 'well-known' theorem in numerical analysis states that

> A number is correct to *at least* J significant digits when its relative error is not greater than 0.5 x 10^{-J}.

To determine J for our density fit, we set

$$E_\rho = 0.5 * 10^{-J}$$

and solve:

$$J = log_{10}\left(\frac{0.5}{E_\rho}\right) \tag{36}$$

A plot of J *vs.* time is given in Fig. 4. (Of course, we could have plotted the integer part of the J expression, but a smooth curve seemed more attractive.) The plot indicates that the fitted density is at its most precise near the start and end of the flight (that is, at the lower altitudes, as we might expect), but even at the higher altitudes there is never less than about four-digit agreement.

The temperature match is more easily expressed. We see from Table 3 that for this test run,

$$T_{gen} - T_{fit} = -6.5H - (-6.507H) = 0.007H \qquad degK$$

where H is in kilometers. Thus at an altitude H of, say, 4 km, the temperature error is only 0.028 out of $(288.15 - 6.5 * 4 = 262.15)$ deg K.

X. Conclusions

The close fit that we obtained seems to us (perhaps in our naiveté) a remarkable result. The generated position values (Figs. 1-3) obviously contained within themselves a core of data sufficient to allow an excellent reconstruction of the air density, sound speed and winds along the trajectory.

Of course, the authors mustn't become so captivated by their results that they overlook the greatly simplified conditions under which those results were obtained. Much work remains to be done. For example, future extensions of this report might consider:

- trajectories that cover more than one altitude zone, so that the generated density and temperature can take take different forms at different times in the same trajectory;
- a series of time intervals, where the density and temperature are known only for the first interval;
- winds that are nonlinear with altitude within each time interval;
- random noise added to the data: to the position values, to the aerodynamic coefficient values, etc.;
- different given trajectory data (say, slant range and slant velocity, rather than X_1, X_2, X_3);
- the complete six degrees-of-freedom equations of motion, rather than the modified point-mass equations.

The possibilities for making conditions more difficult for our program are all too numerous. Still, we have made a start.

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List of Symbols

A	$\left(\frac{\rho S \ell}{2m} C_D\right)\left(\frac{V}{\ell}\right)$, [1/s]
AZ	azimuth of the 1-axis, measured clockwise from North
B	$k_a^{-2}\left(\frac{\rho S \ell}{2m}\right)\left(\frac{V}{\ell}\right)^2$, [1/s^2]
$\vec{C}$	Coriolis acceleration, $-2\vec{\omega} \times \vec{U}$
C_D	drag coefficient: $\lvert drag\ force\rvert = (\rho V^2 S/2) C_D$
C_{D_o}, C_{D2}	zero-yaw and yaw-drag coefficients: $C_D = C_{D_o} + C_{D2}\lvert\vec{\alpha}_e\rvert^2$
C_{ℓ_p}	roll damping moment coefficient: $\lvert roll\ damping\ moment\rvert = \pm(\rho V^2 S \ell/2)(\dot{\phi}\ell/V) C_{\ell_p}$
C_{L_α}	lift force coefficient: $\lvert lift\ force\rvert = \pm(\rho V^2 S/2)\lvert\vec{\alpha}_e\rvert C_{L_\alpha}$
C_{M_α}	static moment coefficient: $\lvert static\ moment\rvert = \pm(\rho V^2 S \ell/2)\lvert\vec{\alpha}_e\rvert C_{M_\alpha}$
$C_{N_{p\alpha}}$	Magnus force coefficient: $\lvert Magnus\ force\rvert = \pm(\rho V^2 S/2)(\dot{\phi}\ell/V)\lvert\vec{\alpha}_e\rvert C_{N_{p\alpha}}$
$C_1, \ldots C_5$	fitting parameters defining the wind, Eqs.(23) and (25), and the temperature gradient, Eq.(26)
D	$\frac{h_a(\vec{G} \bullet \vec{V})}{(1+h_a)V^2}$, [1/s]
D_{jk}	$\partial y_j/\partial q_k$, influence coefficients
E	gun elevation
E_ρ	relative error in the fitted air density
F_j	RHS of the fitting equations (30), j = 1,2,...7
g	$\lvert\vec{g}\rvert$
$\vec{g}$	gravity acceleration
g_o	$\lvert\vec{g}\rvert$ at sea level; the STAT value at 45 deg. N. latitude is 9.80665 m/s

g_1, g_2, g_3	flat-earth system components of $\vec{g}$ $\approx -g_o(X_1/R,\ 1-2X_2/R,\ 0)$
$\vec{G}$	$\vec{g}+\vec{C}$, gravity plus Coriolis acceleration
$\vec{G}_A$	$\frac{1}{1+h_a}\left[\vec{G}+\frac{h_L(\vec{G}\times\vec{V})}{(1-h_M)V}\right]$, [m/s^2]
G_1, G_2, G_3	flat-earth system components of $\vec{G}$
h_a	$\frac{h_L^2}{1-h_M} - h_M$
h_L	$k_a^2\left(\frac{C_{L_\alpha}}{C_{M_\alpha}}\right)\left(\frac{\dot{\phi}\ell}{V}\right)$
h_M	$k_a^2\left(\frac{C_{M_{p\alpha}}}{C_{M_\alpha}}\right)\left(\frac{\dot{\phi}\ell}{V}\right)^2$
H	geopotential altitude, Eq.(18)
H_A	value of H at the starting time of a fitting interval
H_B	value of H at the bottom of a STAT altitude zone (Table 1)
I_x	axial moment of inertia
J	number of significant digits in the fitted air density
k_a^2	$I_x/m\ell^2$
ℓ	reference length
L	latitude at the launch point (for Southern Hemisphere firings, replace L by -L)
m	projectile mass
q_k	the seven initial conditions (k = 1,2,...7) and five parameters (k = 8,9,...12) of the fitting equations (30)
Q	constant in the STAT air density formulas, (21) and (22)
R	effective radius of the earth (6 356 766 m)
S	reference area, $\pi\ell^2/4$

STAT	1962 U.S. Standard Atmosphere, Ref. 6
t	time
T	temperature, degrees kelvin
T'	temperature gradient dT/dH; fitting parameter C_5
T_A	temperature at the starting time of a fitting interval, Eqs.(27) and (29)
T_B	STAT temperature at altitude H_B, see Table 1
$\vec{U}$	projectile velocity with respect to the earth
U_1, U_2, U_3	flat-earth system components of $\vec{U}$
V	$\|\vec{V}\|$
$\vec{V}$	$\vec{U} - \vec{W}$, projectile velocity with respect to the air
V_s	speed of sound
V_1, V_2, V_3	flat-earth system components of $\vec{V}$
$\vec{W}$	wind velocity with respect to the earth
W_1, W_2, W_3	flat-earth system components of $\vec{W}$
$\vec{X}$	projectile position with respect to the earth
X_1, X_2, X_3	flat-earth system components of $\vec{X}$ X_1 : down-range X_2 : the height above sea level X_3 : lateral, positive to the right looking down range
y_j	dependent variables of the fitting equations (30), j = 1,2,...7
Z	geometric altitude, Eq.(16)
$\vec{\alpha}_e$	the yaw of repose, Eq.(4)
ρ	air density

ρ_A	air density at the starting time of a fitting interval. Eq.(29)
ρ_B	STAT air density at H_B, see Table 1
$\dot{\Phi}$	axial spin rate
$\vec{\omega}$	angular velocity of the earth
ω_E	$\lvert\vec{\omega}\rvert$
$\omega_1, \omega_2, \omega_3$	flat-earth components of $\vec{\omega}$: $\omega_E(\cos L\ \cos AZ,\ \sin L,\ -\cos L\ \sin AZ)$
$\dot{(\ )}$	$d(\ )/dt$

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